## TREE DIAGRAMS

**EXAMPLE:** An airline offers discounted tickets to customers who buy tickets early (more than 30 days ahead of time). The company has noticed that 60% of its customers take advantage of the "early-bird" fares, while 25% purchase regular fares, and 15% are "last-minute" customers (less than 48 hours before the flight). They have also figured out that the no-show rate among "early-bird" purchasers is 30%, and only 8% among regular fare customers, and 2% among last-minute customers.

Create a tree diagram of this situation:

$P(E) = 0.60$

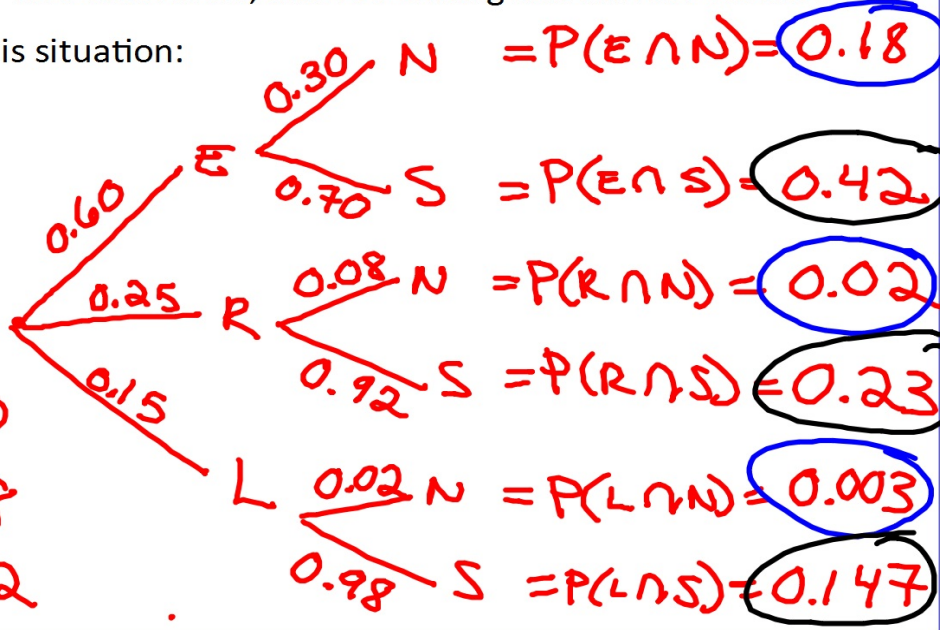
$P(R) = 0.25$

$P(L) = 0.15$

$P(N|E) = 0.30$

$P(N|R) = 0.08$

$P(N|L) = 0.02$



So now let's answer some easy questions:

1) What is the probability of being a no-show (overall)?

$P(N) = 0.18 + 0.02 + 0.003 = 0.203$

2) What is the probability of being a no-show AND a last minute customer?

$P(N \cap L) = 0.003$

3) What is the probability of being a no-show AND an early-bird customer?

$P(N \cap E) = 0.18$

Now let's try some harder ones:

4. Given that you have a no-show in a certain seat, what is the probability of that person being a last-minute customer?

$$0.0148 = P(L \mid N) = \frac{P(L \cap N)}{P(N)} = \frac{0.003}{0.203}$$

5. Given that you have a no-show in a certain seat, what is the probability of that person being a regular-fare customer?

$$P(R \mid N) = \frac{P(R \cap N)}{P(N)} = \frac{0.02}{0.203} = 0.0985$$

6. Given that you have a person show up for a certain seat, what is the probability of that person being an early-bird customer

$$P(E \mid S) = \frac{P(E \cap S)}{P(S)} = \frac{0.42}{0.797} = 0.527$$

Example #1

- A cancer clinic gives free cancer test *(hypothetically)*
- It is known that 2% of the people that come into the clinic have cancer *(hypothetically)*
- It is known the test comes up positive in 98% of people with cancer
- It is known the test comes up positive in 3% of people without cancer

Create the tree diagram:

~~1st event =~~

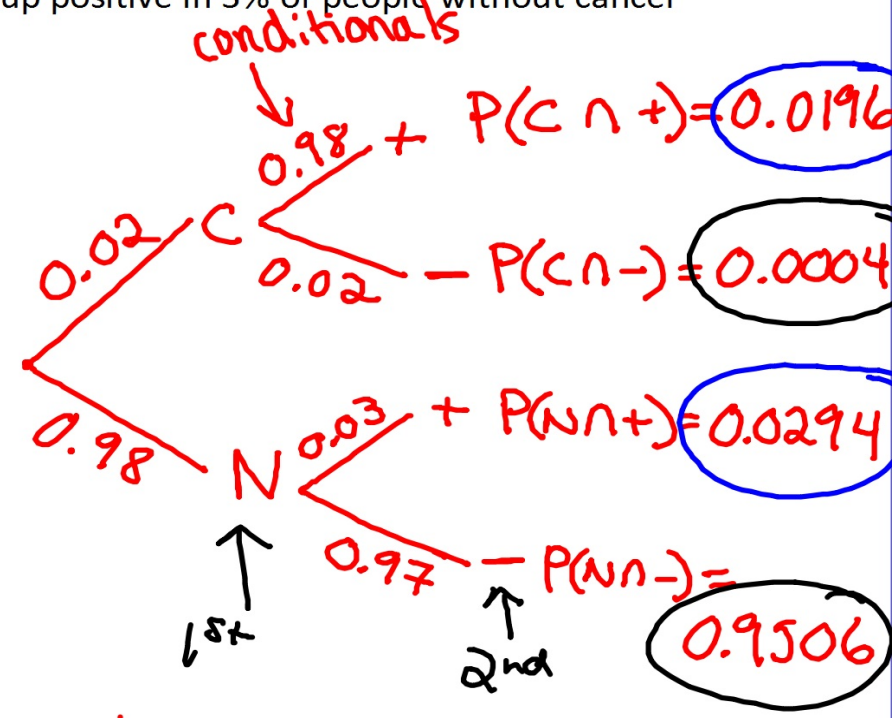
~~2nd event =~~

$P(C) = 0.02$

$P(N) = 0.98$

$P(+|C) = 0.98$

$P(+|N) = 0.03$



Answer the following questions:

1- What is the probability that someone tests positive **given that** they have cancer?

$P(+|C) = 0.98$

2- What is the probability that someone tests positive **given that** they don't have cancer?

$P(+|N) = 0.03$

3- What is the probability that someone tests negative **given that** they have cancer?

$P(-|C) = 0.02$

4- What is the probability that someone tests negative **given that** they don't have cancer?

$P(-|N) = 0.97$

5- What is the probability that someone tests positive? Negative?

$P(+) = 0.049$

$P(-) = 0.951$

6- What is the probability that someone has cancer **given that** they test positive?
*(This is called the accuracy of the test)*

$$P(C|+) = \frac{P(C \cap +)}{P(+)} = 0.40$$

7- What is the probability that someone doesn't have cancer **given that** they test positive?

*(this is called a false positive)*

$$P(N|+) = \frac{P(N \cap +)}{P(+)} = 0.60$$

8- What is the probability that someone has cancer **given that** they test negative?

$$P(C|-) = \frac{P(C \cap -)}{P(-)} = \frac{0.0004}{0.951} =$$

$$4.2 \times 10^{-4}$$

$$0.00042$$

***Example #2:***

- There are 2 textbook making companies, A and B
- It is known that 1% of company A's books are defective
- It is known that 2% of company B's books are defective
- CB South gets 38% of its books from company A and the rest from company B

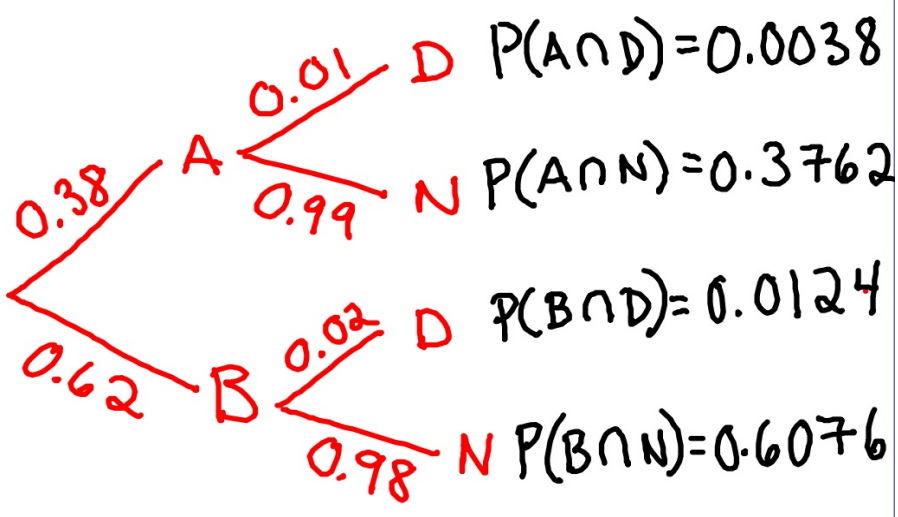
Make the tree diagram below, then answer the questions:

$P(D|A) = 0.01$

$P(D|B) = 0.02$

$P(A) = 0.38$

$P(B) = 0.62$



**Questions:**

1- What is the probability that a book is NOT defective?

$$P(N) = 0.3762 + 0.6076$$
$$= 0.9838$$

2- If a book is not defective, what's the probability that it came from company B?

$$P(B|N) = \frac{P(B \cap N)}{P(N)} = \frac{0.6076}{0.9838} = 0.6176$$

$$P(B|D) = 0.7654$$

3- If we open a book and it **IS** defective, what's the probability that is from company A? Company B?

$$P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{0.0038}{(0.0038 + 0.0124)} = 0.2346$$

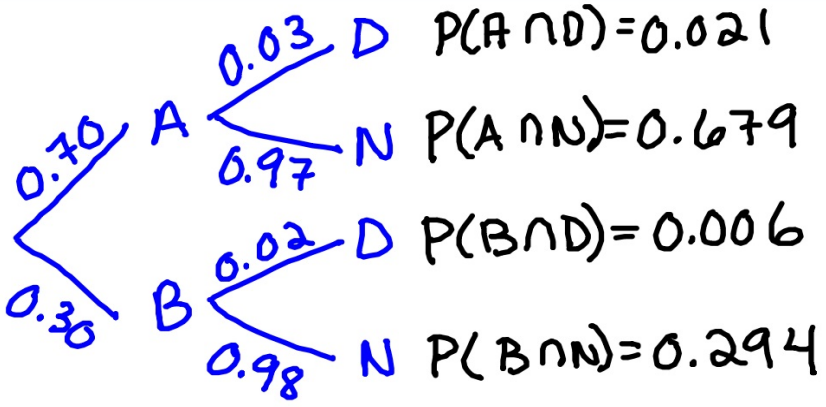
TRY THE PROBLEMS ON THE NEXT FEW PAGES

① $P(A) = 0.70$

$P(B) = 0.30$

$P(D|A) = 0.03$

$P(D|B) = 0.02$




(a) $P(D) = 0.021 + 0.006 = \boxed{0.027}$

(b) $P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{0.021}{0.027} = \boxed{0.7778}$

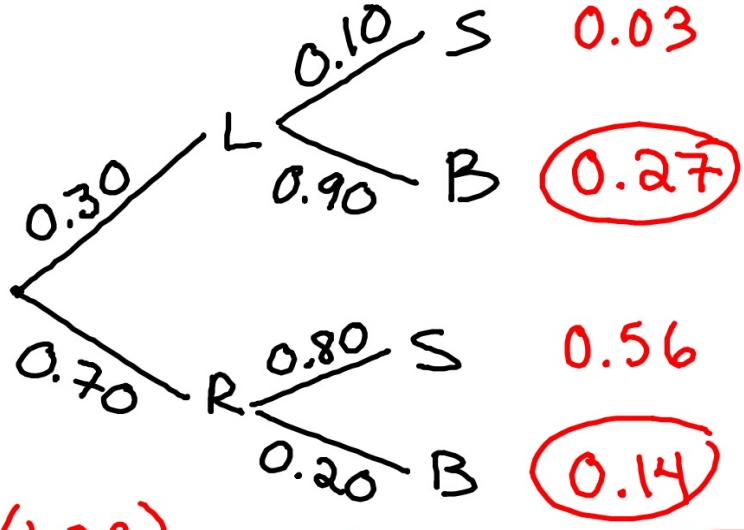
$P(B|D) = \frac{P(B \cap D)}{P(D)} = \frac{0.006}{0.027} = \boxed{0.2222}$

② $P(L) = 0.30$

$P(R) = 0.70$

$P(S|R) = 0.80$

$P(B|L) = 0.90$




$$P(L|B) = \frac{P(L \cap B)}{P(B)} = \frac{0.27}{(0.27 + 0.14)} = 0.6585$$

complete and turn in:

p. 361 #65

Prob. Rules Review #4 (page 1) due tomorrow

I am collecting it!